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Quantification of Mineral Resources

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The term "Mineral Resources" has changed significantly in meaning especially since the turn of the millennium. Until the mid 1990s, a resource simply implied the existence of a mineralisation which might become potentially economic. Nowadays, a Mineral Resource cannot be declared unless there is some justification for expecting the deposit to be profitably exploited as of the date of the study. The term "Reserve" is now applied to a resource which can be proven to be economic through the inclusion of all mining and economic parameters from access to rehabilitation. Since the economic climate is extremely variable, this discussion will concentrate on the "technical" quantification of mineral in situ. The quantification methods discussed here relate to those parameters for which an average (or total) is a suitable estimator. However, the techniques can also be adapted for categorical variables, such as lithology. Some variables from deposits such as oil and gas, which depend on flow characteristics and connectivity, are not suitable for these techniques. A reading list is included and many informative resources are available on the internet (cf. www.kriging.com).

Keywords:

Inverse distance, linear estimation, geostatistics, variogram, kriging, mineral evaluation.

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Introduction

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Mineral deposits are sampled in various ways – geophysical, magnetic mapping and physical sampling such as trenching, drilling, chipping and so on. Samples are collected and various measurements are made on these samples resulting, usually, in a numerical database which contains the location of the samples in additional to chemical assays, visual assessments and geological interpretations. The size of this database can vary from a few dozen measurements to tens of thousands of drillhole "core sections". However, even the largest sampling campaign only measures a minute fraction of the potential mineralisation. For example, the largest diameter drill core currently in use is around 85mm in diameter. If a deposit was drilled on 20m centres, this sampling represents less than 0.015 % of the potential mineralisation.

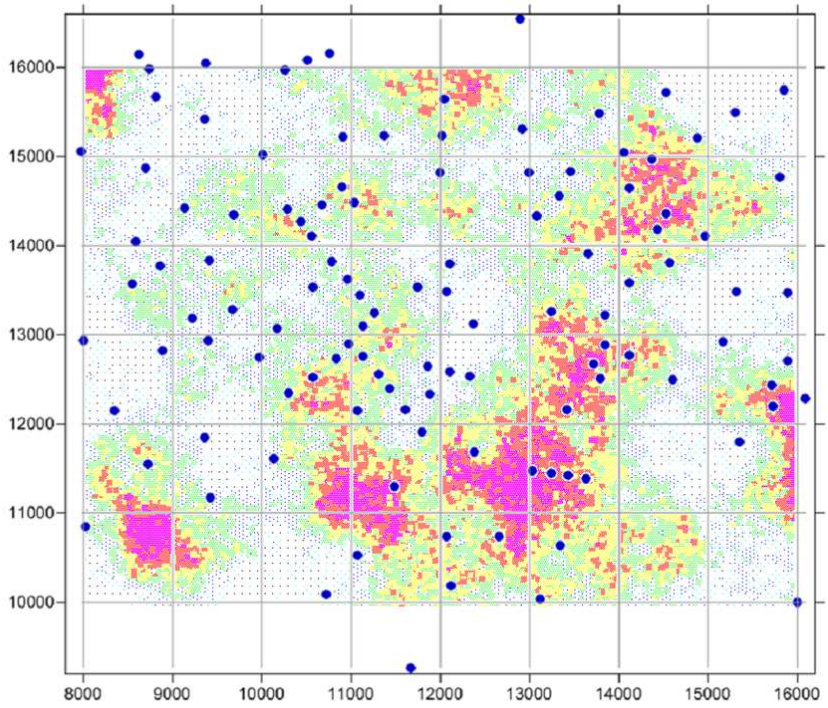

Figure 1: example of sparse sampling in a large deposit

The task of "quantification" is to estimate what potential mineral values might exist at *unsampled locations* based on what has been gathered from the existing sampling. Figure 1

illustrates a hypothetical coal deposit sampled at around 30,000 locations compared to the actual drilling which was carried out comprising 116 drillholes located at sporadic locations across an area of 80 square kilometres. This is a historical data set from a mine in the Karoo region of South Africa. It was first published for teaching purposes by Clark (2000) and can be downloaded from the web site http://www.kriging.com in a simple text format.

To further simplify the discussion, consider any single unsampled location. How could the unknown value at this location be estimated? Before this question can be answered, some assumptions must be made about the deposit, the local mineralisation and the database.

(1) Possibly the most basic assumption is that the samples have been measured precisely. That is, if the sampling process was repeated, the same samples taken again and analysed in the same way, the exact same measurements would appear in the database. This may seem an academic concern but actually has a major influence in the mathematics of some estimation methods which is not immediately obvious to users of software packages. The effect of this assumption may be studied using sampling theory such as that proposed by Gy and Pitard and is subsumed in the "nugget effect" of variography as discussed later in this section.

(2) It is also necessary that the sampling measurements are accurate. That is, they adequately the actual values of the minerals at that location. Again, this may seem a fairly obvious requirement but there are many sampling methods which can introduce bias if not applied correctly. For example, using reverse circulation drilling in a coarse grained base metals deposit can lead to incomplete recovery of mineral in each section of the drillhole – leading to contamination down the hole or, less commonly, actual loss of mineral in the hole.

(3) All software packages – and the mathematics of the estimation methods – depend on the prior geological interpretation of the deposit. It is essential that all major structural features and changes in mineralisation behaviour should be identified. In the parlance of modern geological software packages, homogeneous and physically continuous "domains" should be identified and boundaries put in place. This is generally referred to as "wire framing" but increasingly automated processes are being implemented to provide boundaries for mineralised areas.

(4) All of the currently used mineral resource estimation packages operate on the assumption that there is some sort of relationship between the unsampled values and the values at sampled locations. Space here does not permit discussion of all the possible interpretations of this assumption, so only the most commonly used and widely practiced in mineral resource evaluation will be described in detail.

Distance Weighted Estimation

At this point in the discussion, it must be assumed that the geological interpretation is completed; that structural features have been identified; and that volumes of homogeneous mineralisation type have been domained. The remaining assumption is that the "unknown" value at the unsampled location is somehow related to the values at the sampled locations. There are several ways to interpret this assumption. The most common assumption is that the closest sampled values are most highly related to the unsampled value. Simply stated, the closer

the sample the stronger the relationship between that sample value and the unknown value. Since there are few simple measures of "closeness". Common practice is to calculate the distance between the two locations and to assume that values at larger distance have a weaker relationship. That is, relationship is assumed to be an "inverse" function of distance.

The obvious question, then, is "what function of distance?". Any function which decreases with distance would be a valid inverse distance (ID) function. The most commonly used seem to be inverse distance squared (ID2) and inverse distance cubed (ID3) although straight inverse distance is still occasionally used and there seems to have been a recent fashion for inverse distance to the power of 5 (ID5). This last puts almost all of the weight on the closest sample and produces results similar to those termed "nearest neighbour" or "polygonal" estimators.

Having chosen a suitable function, the calculation as illustrated in Table 1 follows these steps:

(1) Calculate the distance between each sample and the location at which the value is to be estimated;

(2) Calculate the relevant inverse function for that distance;

(3) Standardise these values, one by one, so that they sum to 1. This is to ensure that distance units are irrelevant and that a sensible answer within the range of the data values is achieved;

(4) Multiply the "weights" determined in (3) by the value measured for each sample;

(5) Sum all the value in (4) to obtain the estimated value for the unsampled location.

Table 1: calculation of Inverse Distance Squared (ID2) estimate

Easting (metres)	Northing (metres)	Sample Value				
11739	13246	T				
			(1)	(2)	(3)	(4)
Easting (metres)	Northing (metres)	Sample Value	distance to T	1/ distance^2	weight	weight x Sample Value
11739	13530	9.36	284	0.0000123983	0.4094	3.832
12073	13489	5.2	413	0.0000058615	0.1935	1.006
11262	13246	8.74	477	0.0000043950	0.1451	1.268
11854	12643	4.4	614	0.0000026537	0.0876	0.386
11132	13097	9.78	625	0.0000025598	0.0845	0.827
12371	13126	0.8	643	0.0000024165	0.0798	0.064
			Sum of column:	0.0000302849	1.0000	**7.383**

These results are also illustrated in Figure 2. Using a different ID function will produce a different estimator for the unknown value. Higher powers of distance will weight closer samples more, lower powers will tend to spread the weight out amongst the samples.

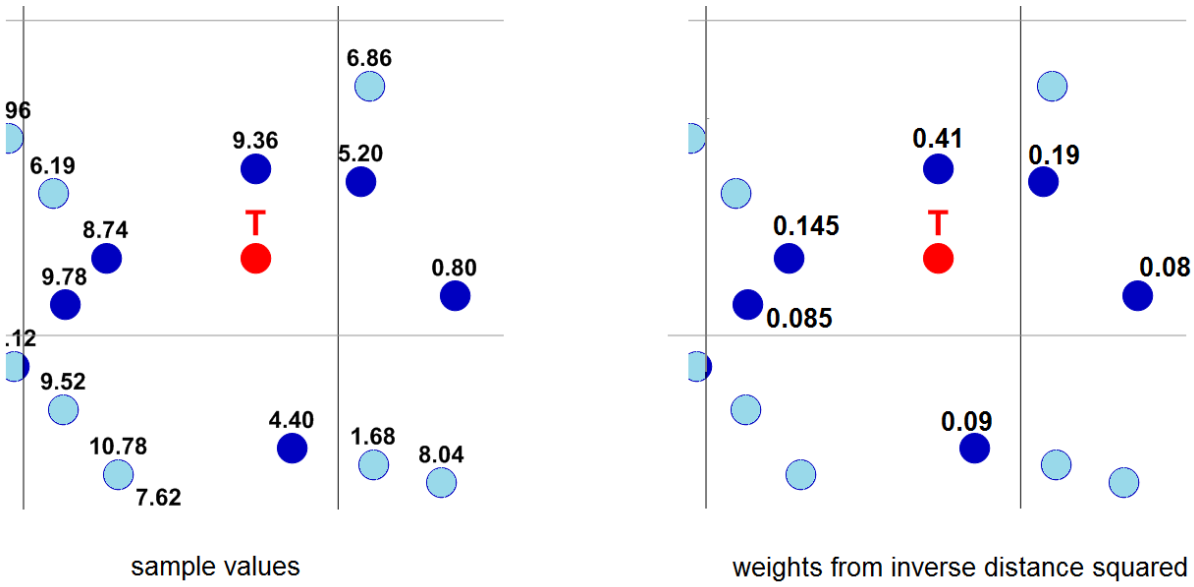

Figure 2: estimation of an unknown value using Inverse Distance squared (ID2)

In this example, 6 samples have been used to provide an estimator. In implementation – for example, when developing macros or software – the operator must supply a maximum number of samples which can be included in the estimation. Also, possibly, the minimum number. Intuitively, more samples would seem to provide a "better" estimator. However, including more samples shifts weight away from the closest samples. Intuitively, there should be a balance between including as many samples as possible and not including too many.

Most software packages will require a "search" radius – the maximum distance at which the relationship is assumed to exist. Or, at least, the maximum distance at which to include samples in the estimation is needed. This is generally determined by empirical assessment of the geological continuity.

Some deposits have preferential directions for continuity of mineral values. Most software packages will allow the introduction of "anisotropy" direction and factors. It should be borne in mind that changing the search radius with direction will not necessarily change how the samples are weighted in different directions. These may require two different specifications in a software package.

One major question remains. How can the choice of appropriate distance function be verified? Which combination of the above parameters is, in some sense, the "best" choice? At the very least, is there some more objective method available to make these choices?

Variography

Distance Weighting estimation methods are based on the assumption that the relationship between values measured at different locations depends on the distance and, possibly, on the relative orientation of those locations.

In the early 1950s, academics and other professionals on the gold mines began to study the nature of these distance/direction relationships. Rather than choosing an arbitrary form for the

relationship they set out to derive the form of the relationship by theoretical or empirical data-driven approaches.

For example, in 1951 Professor de Wijs on sabbatical in South Africa from the Netherlands used mathematical theory to speculate as to how the relationship between gold grades would change with distance based on the assumption that gold grades followed a lognormal distribution. In his work, "relationship" is defined as "successive difference in grade" (Figure 3).

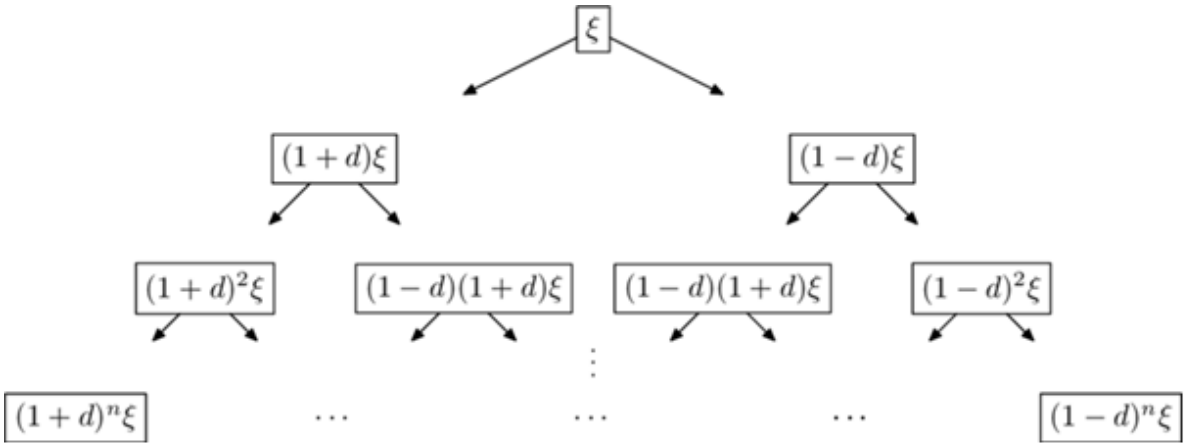

Figure 3: theoretical development of likely successive difference, from de Wijs 1951

Following this lead, a team at Anglovaal Gold including Danie Krige produced tables of covariances between blocks of mined out ground using very large numbers of closely spaced sampling data (Figure 4). These covariances were then used in a standard least squares regression system to predict the likely grade of an as-yet unmined block of ground. This approach was named a "weighted moving average technique" and subsequently became known as "Simple Kriging".

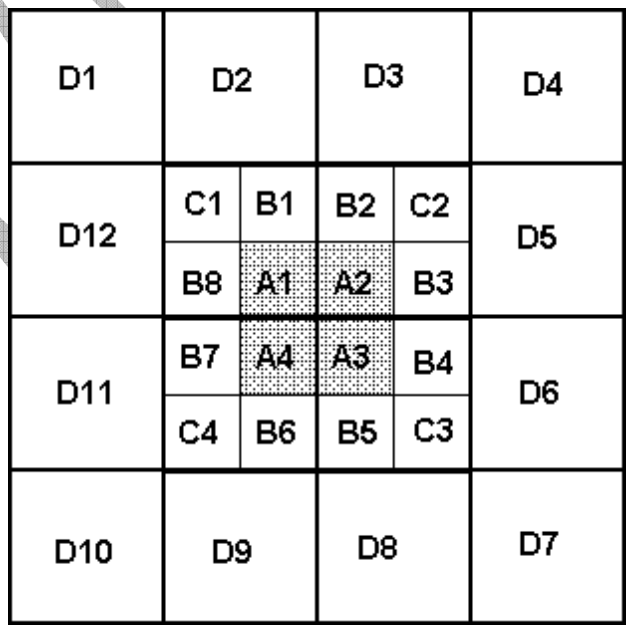

Figure 4: Krige's moving average template for multiple regression estimates

In the mid 1950s, a Doctoral student at the School of Mines in Nancy, France, named Georges Matheron studied these and other distance related approach on a more mathematical basis. His work can be expressed in terms of both covariances and "differences in value". In ideal circumstances, the two definitions of relationship will provide equivalent results. However, the latter requires rather fewer and less rigorous assumptions and tends to be the more commonly used nowadays.

In classical statistics, the covariance is used to measure the one-to-one relationship between two variables. It assumes that the two variables are each Normally distributed or – at least – come from distributions which are symmetrical around their average values. It is also necessary to know – or estimate – the overall average value for the study area (or volume) because this value is used in the calculation of the covariance parameters. In the present context, both "variables" are measurement of the same quantity so that the overall average for both will be the same value. As a consequence of this requirement, this approach to estimation is also known as "Kriging with known mean". Some practitioners take a further step and standardise the covariance by the variance of the sample values, producing a correlation which can vary between +1 (perfect straight-line relationship with positive slope) and -1 (perfect straight-line relationship with negative slope). Zero covariance or correlation denotes no relationship at all between the two values. Calculation of the correlation values also requires that the variance of the measured values to be constant across the study area and to be known or well estimated from the existing data. This is sometimes referred to as "stationarity".

As mentioned previously, de Wijs suggested that the difference between values might provide a useful measure of relationship. In this case, the quantity of interest is simply one measured value minus the other. This introduces the interesting question of whether the calculation should involve the "other" minus the "one". Obviously, either will provide the same result but with the opposite sign. There are several possible solutions to this question, the (perhaps) simplest being to square the difference so that both yield the same result. This is also very tractable for mathematical purposes. It is also intuitively appealing and has the advantage of not requiring the knowledge of overall average or variance of the measured values. An added advantage is that the data do not have to follow a Normal (Gaussian) distribution. The approach is more stable if the difference in value follow a Normal distribution but this is not absolutely necessary in practice. Matheron called this the "Intrinsic Hypothesis" alternatively known as "second order stationarity".

It should be emphasised that if the measured values are "stationary" the two approaches yield identical results. If only second order stationarity is realistic, the "successive difference" approach has greater validity.

To recap, it is assumed that the relationship between values measured at two locations depends on the distance between them and, possibly, their relative orientation. For a specified distance and direction, it is only necessary to find pairs of measured values which are that distance apart and (if necessary) in the same relative orientation to quantify what sort of relationship exists. In practice, a tolerance is allowed so that pairs are found at more or less the same distance and more or less the same direction. When the pairs have been identified, the difference in value is calculated for each pair and squared to obtain a positive figure. The squared values are then averaged to provide a "root mean square" or variance for the difference in value at that distance in that direction. This process is repeated for as many distances and directions as are needed or available.

Matheron suggested that the most appropriate way to display the results would be in the form of a graph showing these calculated variances versus the distance between the measured values. Different symbols may be used to represent different directions, or directional graphs can be shown side by side for comparison. However, Matheron made one modification to this suggestion so that the graph plotted became one-half of the variance versus the distance. This graph is known as the semi-variogram although recently it is more commonly just called a variogram. The most plausible explanation for the use of the semi-variance is that it is directly related to the covariance for the same set of pairs of data:

Covariance = statistical variance of sampled values – semi-variance

If the assumption of a distance based relationship is appropriate, the variance of differences for shorter distances should be less than the variance at greater differences. This should provide a graph which rises as distance increases. In the ideal case, the graph will level off at a certain distance indicating that there is no longer a relationship between values further apart. This distance is usually referred to as a "range of influence" and is generally taken as the maximum search distance for estimation purposes. The height at which the graph levels out is known as a "sill" and should be interpreted as an estimate for the statistical variance of the measured sample values. This quantity may also be estimated using the classical statistical approach. However, it should be borne in mind that the classical estimation approach assumes that the measured values are *not related to one another*. Thus, the sill and the statistical variance of values should be treated as *two different estimates* of the same overall variance in measured value.

In most practical cases, it is not possible to provide "points" on the semi-variogram graph for every possible distance and direction which might occur during the estimation process. This is especially true for regularly spaced (gridded) data where only certain distances are available between samples. The graph produced from the data – sometimes known as an experimental semi-variogram – may contain a limited number of discrete points which will be scattered either side of a general shape for the actual "distance function" which is needed for the estimation process. Common practice is to "fit" a mathematical function to this graph to approximate the appropriate distance function for the data set under analysis (Figure 5).

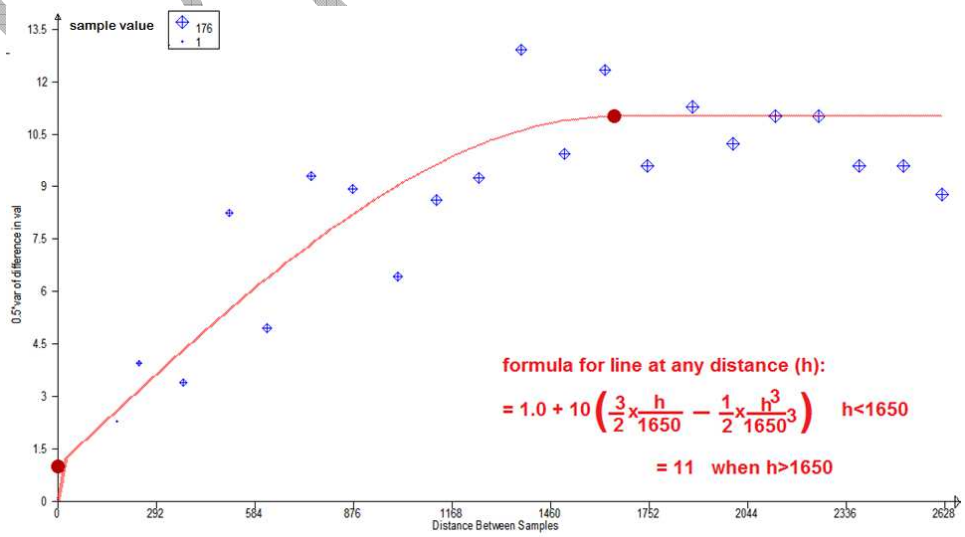

Figure 5: example of calculated semi-variogram (symbols) and fitted model (solid line)

This is known as a model semi-variogram. Studies have been carried out to find general functions which could be applied as models. However, most geological software packages use a small number of proven model types which are mathematically tractable and stable in practice.

It is common, when fitting a semi-variogram model, to find that the model tends to intercept the semi-variance axis rather than falling to zero variance at zero distance. That is, the difference between two sample values very close together is not zero no matter how close the samples are. For example, if a drill core was cut in half and both halves analysed, the measurements on each half would differ. These differences may comprise sampling errors, laboratory variation and/or the unpredictable nature of the mineralisation itself. The magnitude of the semi-variance at (almost) zero distance is called the "nugget effect" or sometimes the "nugget variance". Software packages differ according to whether this nugget effect is used as the semi-variance at zero distance or whether the model is forced to fall to zero at zero distance.

One of the reasons for using the semi-variogram approach rather than the covariance calculation is that the nugget effect is plainly visible in the semi-variogram but not obvious in the covariance graph.

If the distance relationship changes with direction, this is referred to as "anisotropy" and separate model semi-variograms should be fitted for the direction with maximum range of influence and that with minimum range of influence. If working with 3 dimensional data, an intermediate axis will also be required. Most software packages assume that anisotropy follows an elliptical (ellipsoidal) behaviour with the axes being orthogonal to one another. It is usual to model the same nugget effect and the same final sill for all directions.

Deviations from the ideal behaviour of semi-variograms may be diagnostic of violations of the basic assumptions of distance relationships. For example, if the final sill varies with direction, this may be indicative of structural geological features which have not been included in the domaining of the deposit. Taking pairs across a fault line or transgressing across an oxidation/fresh rock boundary are simple examples of non-homogeneity in geology.

Alternatively, such behaviour can occur if data values are very highly skewed and/or the data set contains extreme "erratic" values. This can also produce semi-variogram graphs which look closer to a shotgun scatter than a visible distance function. This can easily be verified by transforming the data values to a more stable behaviour such as a Normal (Gaussian) form. Other transforms, such as an Indicator (0,1) or a rank order transform can be useful in determining what shape is appropriate in some cases. Another common practice, is to remove extreme values from the data set (top cutting) or replacing them by some more acceptable value (capping).

If there is no actual change in relationship with distance the semi-variogram graphs will simply consist of points varying around a constant flat line. In this case, distance weighting is obviously inappropriate as an estimation technique and alternative methods should be sought.
In summary, the justification for a distance weighted estimator is emphasised and quantified by calculating, interpreting and modelling semi-variograms for each geological zone, each mineralisation type and for each measured parameter. It is common practice to use the "variography" to determine search radii and directional factors and to revert to inverse distance type estimation methods to quantify or map mineral resources. Matheron undertook to derive a mathematical approach which would use the semi-variogram models to provide an objective optimal estimation method.

Kriging

Many practitioners in the evaluation of mineral resources carry out a variography exercise to quantify directional controls and ranges of influence for each mineral and each zone. However, a significant proportion of those practitioners then revert to using distance weighting estimators using the arbitrary Inverse Distance functions to determine the weighting factors.

Matheron, Krige and many other authors, however, took the further step and used the established model relationships to produce objectively optimal estimation methods. In geological and mining applications, practitioners follow the estimation techniques developed from Matheron's early work. Mathematical background is extensively published elsewhere and only a brief discussion will be presented here.

The objective, as stated previously, is to estimate the likely mineral values at locations which have not yet been sampled. The most commonly used estimation method is one in which the neighbouring samples are combined in a weighted average where the weights are chosen according to the relationship between the known measured value and the expected value at the unsampled location. This relationship is assumed to be some function of the distance and (possibly) the relative orientation of the locations of sampled and unsampled locations. In traditional Inverse Distance type weighting, the relationship is assumed to be some simple inverse function of distance. This approach can be enhanced somewhat by a variography study as discussed in the previous section.

The term "estimated value" carries with it the implication that this is not the "actual" value which would be measured if a sample was to be taken at the currently unsampled location. That is, there is a difference between the estimated value and the actual value. This difference may also be termed the "error of estimation". Krige developed a multiple regression estimator based on classical Least Squares techniques to determine the weights which would minimise the estimation errors. Covariances were calculated using very large data sets from mined out areas in the gold reefs. For this approach, the average value over the local area must also be known or, at very least, well estimated. This approach has come to be known as "Simple Kriging" or "Kriging with known mean". Matheron developed a similar system modified slightly by the introduction of an additional factor which ensures that estimates are unbiassed without prior knowledge of the overall average of the deposit. This additional factor is known as a "lagrangian multiplier". This approach is generally referred to as "Ordinary Kriging" or "kriging with unknown mean".

In both of these cases – and other similar techniques – development of the mathematical theory reveals that the magnitude of the error incurred in an estimation does not depend only on the relationship between each sample value and the unsampled value. The magnitude of the estimation error also depends on the relationships between the values at the sampled locations.

That is, the magnitude of the estimation error depends on:

(1) how strongly the sampled values are related to the unknown unsampled value;

(2) how strongly the sampled are related to one another;

(3) what weights are allocated to each sampled value.

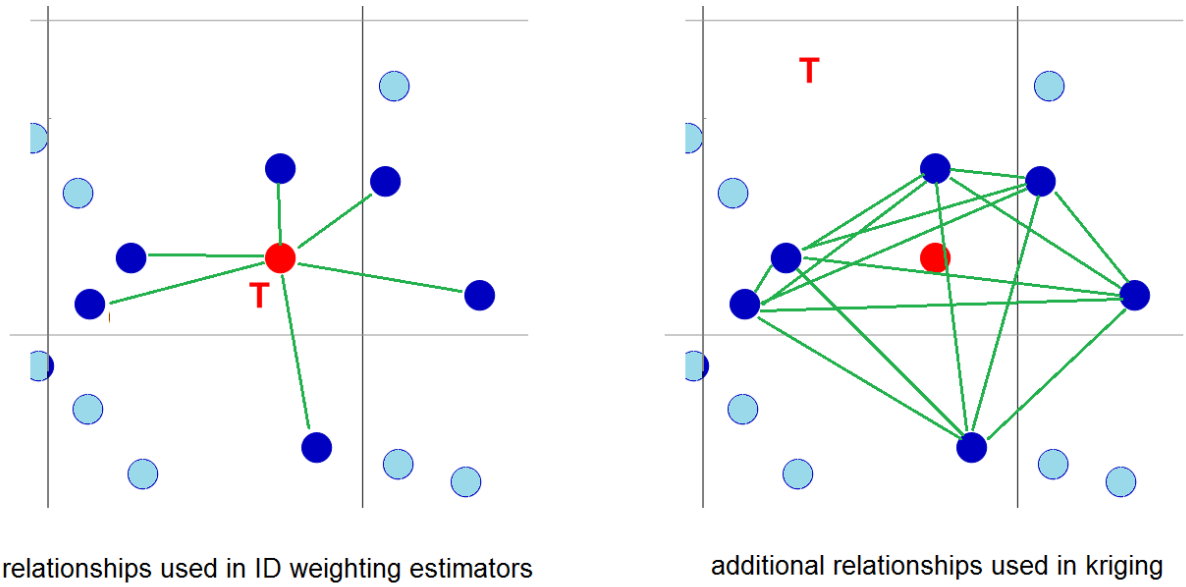

Figure 6: relationships included in the kriging system

To minimise the magnitude of the errors, the weights are chosen such that the sample/unknown relationships balance against the sample/sample relationships (Figure 6). The "lagrangian multipler" used by Matheron allows for the fact that the two factors are unlikely to balance exactly and reflects, to some extent, the efficiency of the sampling layout.

In practice, the Kriging methods reduce to a set of simultaneous equations with a table of the sample/sample relationships – semi-variogram values or covariances for the relevant distance and direction – as the left hand side of the equations. The right hand side of each row in the table is the relationship (semi-variogram or covariance) between the relevant sample and the unsampled value. All of these relationships being calculated using the semi-variogram (or covariance) model for the corresponding distance and direction between the locations of interest.

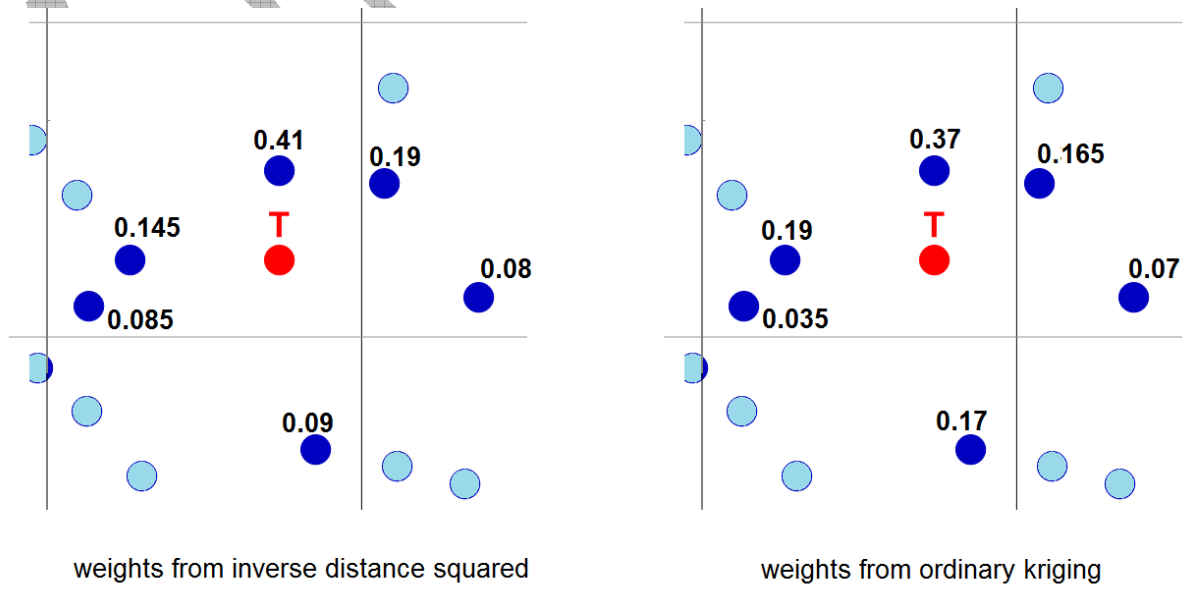

Figure 7: comparison of weights from inverse distance squared and those from ordinary kriging

In Ordinary Kriging, an additional row and column is added to the left hand table for the lagrangian multiplier to ensure a locally unbiassed estimator. In Simple Kriging, the resulting estimator is modified by the overall regional mean to provide an unbiassed estimator. The simultaneous equations may be solved to provide the weighting factors which minimise the magnitude of the difference between the estimated value and the (unknown) actual value at the unsampled location. These weights will differ from a simple ID estimate because the kriging system includes the inter-sample relationships (Figure 7).

The magnitude of the estimation error is sometimes referred to as the "estimation variance". The square root of this quantity represents the standard deviation of the estimation error, often abbreviated to "standard error" which may be seen as a measure of reliability of the estimated value.

It is generally assumed that the error of estimation will follow a Normal (Gaussian) distribution so that classical statistical methods can be applied for confidence levels. This assumption and the general validity of the Kriging method can be verified by a cross validation exercise. This is a diagnostic study in which the estimates and their theoretical errors are compared with actual values where available. The process is as follows:

(1) a sampled value is removed from the data set and so becomes an "unknown" value;

(2) an estimate for the value at that location is calculated using the neighbouring sample providing an estimator and associated theoretical standard error;

(3) the actual estimation error is calculated by subtracting the actual value from the estimated value (or vice versa);

(4) a standardised error is calculated using the ratio of actual error to theoretical standard error.

(5) The sampled value is returned to the data set and another sample removed from the data set. This process is repeated for each sampled value in turn.

If all analysis has been carried out appropriately – variography and kriging processes – the standardised errors should follow a distribution with average zero and standard deviation of 1.0. In ideal circumstances, this distribution will be Normal (Gaussian). In practice, the average error statistic should be approximately zero and the standard deviation approximately 1. Classical statistical tests cannot be used to assess significant differences from the N[0,1] ideal since the estimation errors are highly inter-correlated. However, the exercise does produce results which allow direct comparisons between estimated and actual values and which can be used to produce histograms or probability plots to visually assess the distribution of the actual estimation errors.

Cross validation also provides a simple technique for the identification of anomalies, outliers, non-homogeneities and other departures from ideal behaviour in individual values and overall characteristics. In particular, serious departures from Normality and/or indications for top cutting or capping values can be verified by a cross validation exercises.

It is worth noting that cross validation provides a fairly limited verification when using drilling data since only the very closed spaced sampling down the hole will be included. In this case,

some practitioners prefer to remove a complete drillhole at stage (1) of cross validation to test inter-drillhole or longer range relationships and estimators.

Although most geological software packages offer data transformation for the variography analysis, few offer the capability to Krige on transforms and back-transform the results. If the data follow a lognormal distribution, Lognormal Kriging or a Lognormal shortcut may be appropriate. If the data is severely non-Normal but not lognormal, a Multi-Indicator transformation may be the most appropriate option. Cross validation has been used to compare Kriging methods since the 1970s.

It has been said previously that authors other than Krige and Matheron were producing similar estimation methods in the 1950s and 1960s. However, Krige and Matheron focussed their work not only on the production of contour maps but also on the estimation of values into mining blocks. In fact, Krige's early works were completely oriented to block estimation. Matheron developed his methodology on "point" sampling but extended his theory into the estimation of volumes of ground. Although the mathematical background is highly complex, the practical applications of block kriging are relatively straightforward. Without going into the mathematics, the intuitive extension of kriging to block estimation is simple:

(1) The left hand table of sample/sample relationships remains unchanged;

(2) The right hand column remains as the sample/unknown relationship but demands that the relationship between a sample and a block of ground be quantified;

(3) An additional component to the estimation variances comes into play to allow for the size, shape and orientation of the block.

In practice, the block or volume to be estimated is considered to be an aggregate of many sample-sized units. That is, a block is equated to a large number of individual samples. In Matheron's theoretical approach, the number of samples is practically infinite. In practice, a finite and relatively small number of sample "points" is used. This is sometimes referred to as "discretisation". The relationship between a single sample and the average value of a block is found by pairing the single sample with each point inside the block in turn and averaging all the resulting semi-variogram values. The component (3) above is found by pairing every point in the block with every point in the block, calculating the semi-variogram value for each pairing and then averaging all of these relationships.

It should be fairly obvious that the most important parameter in this process is simply: how many samples adequately represent a given volume of ground? Most software packages operate with a default "discretisation" which is generally a very small number – for example, 3 by 3 by 3 or 5 by 5 by 1. This question has been discussed in the literature since the mid 1970s (cf. Clark 1976)

The weights for each sample will differ depending on the size, shape and relative orientation of the block (Figure 8). This approach, sometimes known as "direct kriging", produces estimates for block values which should be more reliable than any Inverse Distance weighted average at the block centre.

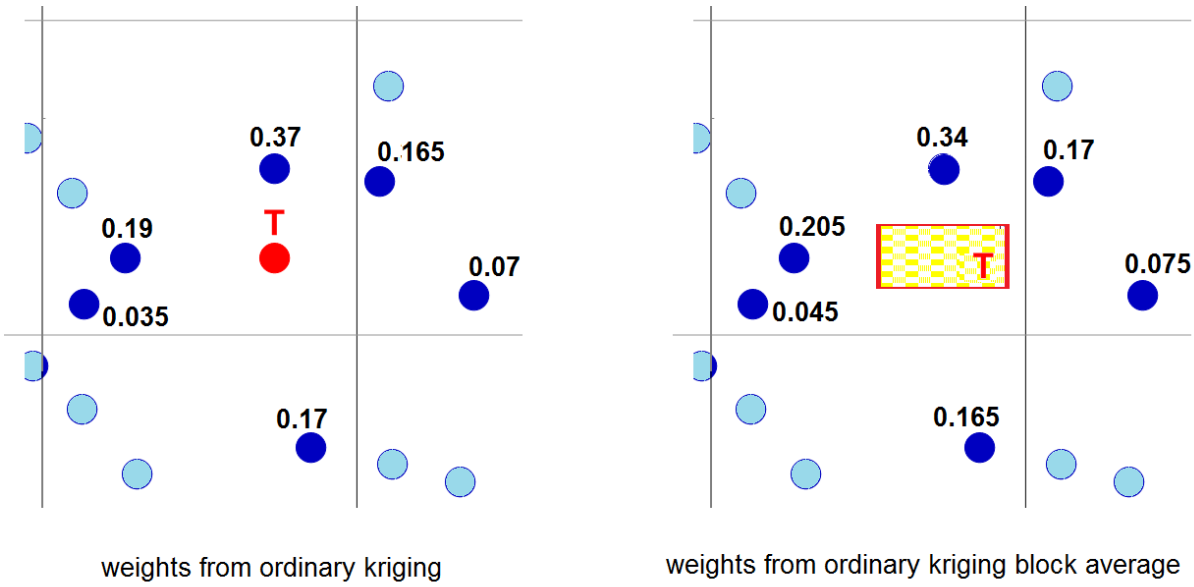

Figure 8: comparison of weights from ordinary kriging when estimated a new borehole and the average grade over a block

Summary

In summary, variography provides a method of quantifying the geological continuity of mineral values within a given domain for a given measured variable. The efficiency of Inverse Distance style weighting methods can be improved significantly if combined with variography. Kriging uses the geological model as quantified in the variography to produce optimal estimates for values at unsampled locations and/or blocks of ground with the added advantage of an objective measure of the confidence which might be placed on these estimates.

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